

E4: $\int_{-1}^1 3x^2 \sqrt{x^3+1} dx$

WE NEED U-SUB!

$u = x^3 + 1$
 $du = 3x^2 dx$

$\int_0^2 u^{1/2} \cdot \frac{du}{3x^2}$

CHANGE LIMITS:
 $u = x^3 + 1 \Rightarrow \text{AT } x=1 \Rightarrow u=2$
 $u = x^3 + 1 \Rightarrow \text{AT } x=-1 \Rightarrow u=0$

$\int_0^2 u^{1/2} du$
 $\left[\frac{2}{3} u^{3/2} \right]_0^2$
 $\frac{2}{3} [2^{3/2} - 0] = \boxed{\frac{2}{3}(2\sqrt{2})}$

~~$\int_0^1 (2x-3)^5 dx$~~

$u = 2x-3$
 $du = 2 dx$

$\int_{-3}^{-1} u^5 \frac{du}{2}$

$\frac{1}{2} \int_{-3}^{-1} u^5 du$
 $\left[\frac{1}{12} u^6 \right]_{-3}^{-1} = \frac{1}{12} ((-1)^6 - (-3)^6)$
 $= \frac{1}{12} (1 - 729) = \boxed{-\frac{182}{3}}$

~~$\int_0^{\pi/6} \cos^4 3x \sin 3x dx$~~

$u = \cos 3x$
 $du = -3 \sin 3x dx$

$x=0 \Rightarrow u=\cos 0=1$
 $x=\pi/6 \Rightarrow u=\cos \pi/2=0$

$-\frac{1}{3} \int_1^0 u^4 du$
 $\frac{1}{3} \int_0^1 u^4 du$
 $\left[\frac{1}{15} u^5 \right]_0^1 = \boxed{\frac{1}{15}}$

$\int_{\pi/4}^{\pi/2} \cot \theta \csc^2 \theta d\theta$

$u = \cot \theta$
 $-du = -\csc^2 \theta d\theta$

$x=\pi/4 \Rightarrow u=\cot \pi/4=1$
 $x=\pi/2 \Rightarrow u=\cot \pi/2=0$

$-\int_1^0 u du$
 $\int_0^1 u du$
 $\left[\frac{1}{2} u^2 \right]_0^1 = \boxed{\frac{1}{2}}$